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# OPTIMUM FLOW DISTRIBUTION IN THE NETWORK WITH ADAPTIVE DATA TRANSFER

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***Abstract—****This paper considers a fundamentally new model of data network construction for servicing the flows given by the gravitational matrix with restrictions on the time of information transmission. The proposed network model differs from the known ones because it has an expanded capacity of node and channel resources, combining the lowest possible cost of channels and switching nodes. As part of the development of the proposed model and algorithm for allocation of flows in a full mesh network, the developed mathematical apparatus provides a high degree of reliability and survivability of the synthesized network as a whole. One of the used approaches to the solution of the linear programming problem is based on the choice of the target function, the type of which is determined by the consumer conditions of the synthesis of a particular data network. Within the framework of the article, it is established that the linear programming problem, for each specific case, has an admissible, practically realizable solution with the optimal choice of the target function without additional simplex transformations. An important obtained result of the research is the simplicity of flow control, which is in direct dependence on the strict ordering of the proposed structure, because it was possible to connect by analytical dependences the flows in the branches and the path data transfer flows. The obtained analytical results can be used as a basis for statistical algorithms of information flow control.*

**Index Terms—**Data network; network model; gravity matrix; switching node; path flow; linear programming; target function; static flow distribution; network degradation; adaptive control algorithm.

## I. INTRODUCTION

To meet the needs of consumers, in accordance with the Order of the National Telecommunications Network [1], it is necessary to fully ensure the proper functioning of the set of software (software and hardware), technical means of multimedia communication and data processing system, storage and transmission, as well as cryptographic and technical protection of information designed to ensure the exchange of public information and / or information with restricted access.

The strategic importance of quality development of info communications due to the need to integrate Ukraine into the world global information space; providing unlimited and reliable user access to information resources and special software tools of various networks; creating technological conditions for the mobility of users of telecommunications services in all spheres of social life in a dynamic global environment [2].

According to the given topological structure of the information network, the matrix of input streams and bandwidths it is necessary to find such values of streams that minimize the average delay of messages in the network, that is to solve the optimization problem.

## II. PROBLEM STATEMENT

To solve the problem of building a data transmission network, it is necessary to know the number of switching nodes, their locations and the gravity matrix between them [3]. The resulting network must serve the flows specified by the gravity matrix and have a minimum cost, consisting of the cost of channels and switching nodes.

Restrictions are imposed on the designed network in terms of information transmission time, reliability, survivability, capacity of node and channel resources, as well as conditions for the use of existing fragments of the network.

## III. REVIEW OF PUBLICATIONS

This formulation of the problem, based on the analytical study of the work [4] allows us to obtain a static flow distribution plan on the synthesized network, which can be the basis for static algorithms for information flow control.

An important area of research in the field of information transmission is the analysis and solution of problems related to delay minimization and load distribution using optimization models. Optimization problems are present both in packet and message switching networks and in digital circuit switching networks. The specific implementation of the algorithm depends significantly on the specific features of the network, but in general for different networks a rather similar mathematical apparatus is used – shortest path algorithms and flow algorithms, applied to the flow models of networks based on traffic intensities arriving in the communication lines.

An important factor limiting the technical possibilities for optimal data transmission, according to papers [5], [6], is that in flow models an implicit assumption is made that the statistics of traffic entering the network do not change over time. Such an assumption is justified when these statistics change very slowly compared to the average time required to reduce queues in the network, and when flows in lines are measured by time averaging [6].

However, the task of load balancing based on an optimal, including in the time range, model, which allows to reduce the average delay of packets in the network based on the prediction of traffic intensity on the communication lines containing an arbitrary number of channels, looks very relevant.

## IV. PROBLEM SOLUTION

This formulation of the problem allows us to obtain a model of static distribution of flows of the synthesized network, which can be the basis for static algorithms of information flow control.

Let us represent the network model (a fragment is shown in Fig. 1) in the form of a fully connected graph $G$, consisting of $k$ nodes, two of which are distinguished as a source $S$ and a receiver $t$.

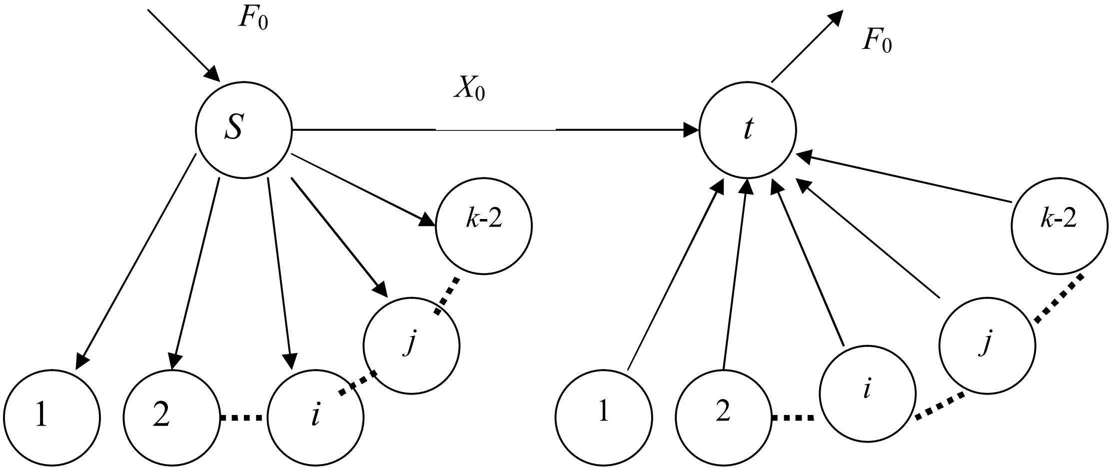

Fig. 1. Fragment of a fully connected network

Let us define the flow in an arbitrary branch $F_{ij}$ to be equal to the sum of all path flows $X_p$ flowing through this branch, i.e.:

$$F_{ij} = \sum_{\text{all the way}} X_p, \quad i,j=\overline{1,k}. \tag{1}$$

The throughputs $V_{ij}$ of the corresponding branches, providing the minimum average delay in the network, can be determined from the solution of the optimization problem in any of the statements [4], [7], [8].

Since the flows $F_{ij}$ satisfy the requirements [7], then it can be stated that the path flows found from the solution of system (1) will also provide the minimum average delay, i.e. they will be optimal.

However, the system of equations (1) is not single-valued, because the number of paths of the network many times exceeds the number of branches $F_{ij}$. This means that in system (1) the number of variables exceeds the number of equations. Since this system is a system of linear algebraic equations, it can be considered as a linear programming problem 8, in which system (1) acts as constraints. It can be solved with an appropriate choice of the target function, for example, by the table simplex method 9.

For any pair of corresponding subscribers S and t it is possible to write the system of equations including $\beta \le k-2$ transit nodes. Given the large number of path streams, let us limit the transfer of information only along the routes containing no more than two transit nodes. Then the total number of $n_{\text{var}}$ variables (path flows $X_p$) according to [4], [7] is determined by the number of placements and will be:

$$n_{trac} = 1 + \sum_{z=1}^{2} A_{\beta}^{z}. \tag{2}$$

Under these conditions, the system of restrictions can be represented as follows:

$$\begin{cases} F_{si} = \sum\limits_{j=1}^{\beta} X_{(i-1)\beta+j}\ , \quad i,j=\overline{l,\beta},\ j>i, \\ F_{it} = \sum\limits_{j=1}^{\beta} X_{[(j-1)\beta+i]}, \\ F_{ij} = X_{(i-1)\beta+j} - X_{(j-1)\beta+i}, \\ F_{st} = X_o. \end{cases} \qquad (3)$$

In accordance with the law of conservation of the flow for each node,

$$\sum_{\alpha=1}^{p+1} F_{i\alpha} = F_o \begin{vmatrix} 1, & i=S, \\ 0, & i\neq S,t, \\ -1, & i=t, \end{vmatrix} \quad i=\overline{1,k}, \qquad (4)$$

where $p$ is the connectivity of the graph.

In system (3), the equations are a linear combination of the others, so we discard any equations from consideration as linearly dependent.

Let us express basic variables in terms of free ones (reduction to canonical form).

$$\begin{cases} X_{[(i-1)\beta+i]} = F_{si} - \sum\limits_{j=i+1}^{\beta} F_{ij} - \left( \sum\limits_{j=1}^{i-l} X_{[(i-1)\beta+j]} - \sum\limits_{j=i+1}^{\beta} X_{[(j-1)\beta+i]} \right), \\ X_{[(i-1)\beta+j]} = F_{ij} + X_{[(j-1)\beta+i]}, \\ X_o = F_{st}. \end{cases} \qquad (5)$$

where $j>i$.

On the right side of the system of equations (3) there are basic variables, the number of which $n_{\text{bas}}$ is equal to the number of equations:

$$n_{\text{bas}} = \frac{(\beta+2)(\beta+1)}{2} - \beta = \frac{\beta^2+\beta+2}{2}. \qquad (6)$$

The total number of variables (path flows) is determined by expression (2), so that the number of free variables

$$n_{\text{trac}} = n_{\text{var}} - n_{\text{bas}} = 1 + \sum_{z=1}^{2} A_{\beta}^{z} - \frac{\beta^2+\beta+2}{2}. \qquad (7)$$

The result of the solution of the linear programming problem depends on the choice of the target function $L(X)$, the type of which is determined by the specific conditions of the general problem of the synthesis of the communication network. As an example, we set the condition: the maximum information from node $S$ to node $t$ is transmitted along the routes, which contain no more than one transit node, i.e.

$$L(X) = X_o + \sum_{i=1}^{\beta} X_{[(i-1)\beta+i]} = X_o + \sum_{i=1}^{\beta} F_{si} - \sum_{i=1}^{\beta} \sum_{j=i+1}^{\beta} F_{ij} - \sum_{i=1}^{\beta} \left( \sum_{j=1}^{i-1} X_{[(i-1)\beta+j]} - \sum_{j=i+1}^{\beta} X_{[(j-1)\beta+i]} \right). \qquad (8)$$

According to the flow conservation law

$$X_o + \sum_{i=1}^{\beta} F_{si} = F_o. \qquad (9)$$

Objective function (8), taking into account (9), will take the form:

$$L(X) = F_0 - \sum_{i=1}^{\beta} \sum_{j=1}^{\beta} F_{ij} - 2\sum_{ij} X_s, \qquad (10)$$

where

$$2\sum_{ij} X_s = \sum_{i=1}^{\beta} \left( \sum_{j=1}^{i-1} X_{[(i-1)\beta+j]} - \sum_{j=i+1}^{\beta} X_{[(j-1)\beta+i]} \right). \qquad (11)$$

Final target function for solving the problem with the tabular simplex method is converted to the following form:

$$L^{\min}(x) = -L(x) = \sum_{i=1}^{\beta} \sum_{j=1}^{\beta} F_{ij} - F_0 - 2\sum_{ij} (-X_s) \to \min \qquad (12)$$

If the conditions are met

$$\begin{cases} F_{si} - \sum\limits_{j=i+1}^{\beta} F_{ij} > 0, \\ F_{ij} > 0, \end{cases} \qquad (13)$$

the linear programming problem has a feasible solution.

Furthermore, if the path flows forming the free variables $X_{fr}$ are oriented in the direction from node $S$ to node $t$, then the linear programming problem also contains an optimal solution, which without additional simplex transformations is found by zeroing the free variables ($X_{fr}=0$):

$$\begin{cases} X^{*}_{[(\text{i-1})\beta+i]} = F_{si} - \sum\limits_{j=i+1}^{\beta} F_{ij}, \\ X^{*}_{[(i-1)\beta+j]} = F_{ij}. \end{cases} \qquad (14)$$

$$L^{\min}(X) = \sum_{i=1}^{\beta} \sum_{j=1}^{\beta} F_{ij} - F_0. \qquad (15)$$

To get a complete picture of the distribution of flows in the network, it is necessary to solve a similar problem for each pair of allocated nodes with their own initial flows $F_0^k$. The resulting $F_{ij}^{kl}$ value for each variant of the problem must be summed over all variants $\left(i, j = \overline{1, k}\right)$ of the solution and can be regarded as the resulting loads on the corresponding link.

If, in this case, the initial flows $F_o^k$ are generated by each node $k$, and do not come from outside, then their values must correspond to the gravitation matrix $\left\|\lambda_{ij}\right\|$, the elements of which, as a rule, are set as initial data.

If the considered structure is a fragment of some global network, its initial flows can be used to combine individual fragments through special gateways into a broader structure, respecting the law of conservation for each pair of adjacent nodes belonging to different fragments.

## V. RESULTS

The relations obtained as a result of this research describe the static distribution of flows in the full-connection network. However, this structure is hardly advisable to implement in practice in the synthesis of data transmission networks because of its extremely high cost, especially in cases where it is not required by the conditions of reliability. The fact that in such a structure, each subscriber can communicate with any other through an independent channel, facilitates the task of routing messages, but is often insufficient in terms of network degradation. Equations (3) provide, in addition, alternative transmission routes on paths containing one or two transit nodes. Because of this, equations (3) can be easily transformed with respect to lossless structures, e.g., by branch exclusion, to obtain structures with a given connectivity.

The easiest way to perform this procedure is to eliminate branches in order to obtain a regular structure with specified properties and satisfying the requirement for reliability (connectivity). For example, if we exclude links between all nodes which form an external Hamiltonian cycle, the connectedness of the graph decreases by two units.

In this case equations (3) will be considerably simplified due to the fact that a part of variables will be reduced to zero:

$$\begin{aligned} &F_{st} = 0, \\ &F_{s1} = \sum_{j=1}^{\beta} X_j = 0, \\ &F_{\beta t} = \sum_{j=1}^{\beta} X_{\beta j} = 0, \\ &F_{i(i+1)} = X_{(i-1)(\beta+1)+2} - X_{i(\beta+1)} = 0. \end{aligned} \qquad (16)$$

As $F_{s1}, F_{\beta t}, F_{i(i+1)}$ turns to zero, so do all path flows $X_j, X_{\beta j}, X_{[(i-1)(\beta-1)+2]}$, $X_{i(\beta+1)}$, which together form the corresponding flows in the branch. After such changes, it is necessary to require that the law of conservation of flux be satisfied. Since the number of path flows (variables) is drastically reduced, the remaining system of equations turns out to be one-valued, i.e. it has a single solution.

In practice, there is an independent task of flow control, associated with the development of adaptive control algorithms that respond both to changes in the flows outside the nominal values, and to dynamic changes in the state and performance of network elements. At the same time, certain difficulties arise, the reason for which is that in dynamic algorithms, the processes describing the system behavior depend on the decisions made, and these decisions must take into account the current state of the network. This means that each initial condition will correspond to its own optimal distribution of flows, and for optimal routing it is necessary to solve the optimization problem under new initial conditions each time, which not only requires knowledge of the network state at each current moment, but also cannot be performed in real time.

This independent problem, which is not related to topological design, must be solved by applying an advanced control system that maintains the harmonious internal organization of the network. The goal of static control is to keep traffic within limits compatible with available resources at transmission rates that are close to nominal. An important factor is the simplicity of flow control, which is in direct dependence on the strict ordering of the structure, because in this case it is possible to link branch flows and path flows by analytical relationships.

## VI. CONCLUSIONS

The minimum throughput of a branch in the corresponding route is determined by the path flow, and there is always a branch in which there is one path flow, the size of which is determined by the throughput of this branch.

Path flows can be considered optimal, and the resulting static distribution of flows can be regarded as a variant of optimal routing.

With a static distribution of flows, a properly designed network should itself prevent congestion without the participation of a specially organized control system that coordinates flows throughout the entire distributed system.

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**О. В. Шефер, Фрхат Алі Алнаері. Оптимальний розподіл потоків в мережі з адаптивною передачею даних**

У роботі розглянута принципово нова модель побудови мережі передачі даних для обслуговування потоків заданих матрицею тяжіння з обмеженнями по часу передачі інформації. Запропонована модель мережі відрізняється від відомих тим, що має розширену ємність вузлових і канальних ресурсів, поєднуючи в собі мінімально можливу вартість каналів і вузлів комутації. У рамках розробки запропонованої моделі та алгоритму розподілу потоків у повнозв'язній мережі, розроблений математичний апарат забезпечує високу ступінь надійності й живучості в цілому, мережі, що синтезується. Один із підходів, що використовується для вирішення завдання лінійного програмування, заснований на виборі цільової функції, вид котрої визначається споживчими умовами синтезу конкретної мережі передачі даних. У рамках статті встановлено, що задача лінійного програмування, для кожного конкретного випадку, має допустиме, практично реалізоване рішення при оптимальному виборі цільової функції без додаткових симплекс-перетворень. Важливим отриманим результатом досліджень є простота управління потоками, котра знаходиться в прямій залежності від чіткої впорядкованості структури, що запропонована, оскільки вдалося зв'язати аналітичними залежностями потоки в гілках і шляхові потоки передачі даних. Отримані аналітичні результати можуть бути покладені в основу статистичних алгоритмів управління потоками інформації.

**Ключові слова:** мережа передачі даних; модель мережі; матриця тяжіння; вузол комутації; шляховий потік; лінійне програмування; цільова функція; статичний розподіл потоків; деградація мережі; адаптивний алгоритм керування.

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**А. В. Шефер, Фрхат Али Алнаэри. Оптимальное распределение потоков в сети с адаптивной передачей данных**

В работе рассмотрена принципиально новая модель построения сети передачи данных для обслуживания потоков заданных матрицей тяготений с ограничениями по времени передачи информации. Предложенная модель сети отличается от известных тем, что имеет расширенную емкость узловых и канальных ресурсов, сочетая в себе минимально возможную стоимость каналов и узлов коммутации. В рамках разработки предложенной модели и алгоритма распределения потоков в полносвязной сети, разработанный математический аппарат обеспечивает высокую степень надежности и живучести синтезируемой сети в целом. Один из используемых подходов к решению задачи линейного программирования основан на выборе целевой функции, вид которой определяется потребительскими условиями синтеза конкретной сети передачи данных. В рамках статьи установлено, что задача линейного программирования, для каждого конкретного случая, имеет допустимое, практически реализуемое решение при оптимальном выборе целевой функции без дополнительных симплекс-преобразований. Важным полученным результатом исследований является простота управления потоками, которая находится в прямой зависимости от строгой упорядоченности продложенной структуры, поскольку удалось связать аналитическими зависимостями потоки в ветвях и путевые потоки передачи данных. Полученные аналитические результаты могут быть положены в основу статистических алгоритмов управления потоками информации.

**Ключевые слова:** сеть передачи данных; модель сети; матрица тяготений; узел коммутации; путевой поток; линейное программирование; целевая функция; статическое распределение потоков; деградация сети; адаптивный алгоритм управления.

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