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STRUCTURAL-PARAMETRIC DESIGN OF ROBUST CONTROL SYSTEM WITH REDUCED-ORDER OBSERVER

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Abstract—*The paper dials with the features of digital robust control system with reduced-order observer design. Optimal control law robust optimization was performed. The result obtained for lateral channel of small unmanned aerial vehicle.*

Index terms—Unmanned aerial vehicle; optimal control law; robust control; reduced-order observer.

I. INTRODUCTION

In recent years unmanned aerial vehicle (UAV) are used in different fields of human endeavours. Performed tasks require certain control accuracy and the quality. Therefore, the synthesis of an effective control system is an actual task.

Optimal controller synthesis can be performed mostly only for the systems with the full state space vector measurements. Otherwise, the full state space vector has to be restored. For this purpose it is possible to use stochastic optimal observer (Kalman filter). But it twice increases the order of control law [1], [3]. Therefore advisable to consider an alternative, restore the full vector of the system using a reduced-order observer (Luenberger observer).

A significant influence on such control system quality and efficiency has a selection of reduced order observer eigenvalues. To simplify this procedure, the choice is proposed to carry out for a continuous system. The synthesized observer converted to the discrete form.

Good results can be achieved if the robust optimization of obtained optimal result will be performed.

II. ROBUST CONTROL LAW SYNTHESIS

The continuous optimal reduced order observer is the first step of the control systems synthesis. The next step is to convert it into the discrete form.

This is due to the fact that during synthesis it is necessary to determine the eigenvalues of the Luenberger observer. It is difficult, if a digital system is considered. The stability condition of digital system is the location of the eigenvalues in the single radius circle. The stability condition of continuous systems is the location of the eigenvalues in the left half-plane. Therefore it is easier to solve the problem of eigenvalues selection.

The initial data for the reduced order observer synthesis are the state space matrix of the system [1], [2], [3].

$$\mathbf{x} = \mathbf{Ax} + \mathbf{Bu};$$
$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}.$$

As the number of measurements is less than the number of phase coordinates, it is necessary to define the filter to minimize the error rate $\varepsilon = x - \hat{x}$. Choose a variable $\mathbf{p}(t)$ that is a measure of variables that are not observed $\mathbf{p} = \mathbf{C}'\mathbf{x}$, where $\mathbf{C}'$ is the matrix of variables that must be reconstructed. Then from the relation:

$$\mathbf{y} = \mathbf{Cx};$$
$$\mathbf{p} = \mathbf{C}'\mathbf{x},$$

follows that the full state of the system $\hat{\mathbf{x}}$ is described by the expression

$$\hat{\mathbf{x}}(t) = \begin{pmatrix} \mathbf{C} \\ \mathbf{C}' \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{y}(t) \\ \mathbf{p}(t) \end{pmatrix}.$$

It is convenient to write the previous expression in the following form:

$$\begin{pmatrix} \mathbf{C} \\ \mathbf{C}' \end{pmatrix} = (\mathbf{L}_1, \mathbf{L}_2),$$

so that $\hat{\mathbf{x}} = \mathbf{L}_1\mathbf{y} + \mathbf{L}_2\mathbf{p}$.

The variable $\mathbf{p}$ can be found, based on the following differential equation:

$$\dot{\mathbf{p}} = \mathbf{C}'\mathbf{Ax} + \mathbf{C}'\mathbf{Bu} \text{ or } \dot{\mathbf{p}} = \mathbf{C}'\mathbf{AL}_2\mathbf{p} + \mathbf{C}'\mathbf{AL}_1\mathbf{y} + \mathbf{C}'\mathbf{Bu}. \quad (1)$$

In this equation $\mathbf{y}$ is the control variable.

To synthesize the reduced order observer without defining derivatives (necessary to obtain additional information), suppose that

$$\mathbf{q} = \hat{\mathbf{p}} - \mathbf{Ky}. \quad (2)$$

The equations (1) and (2) show that $\mathbf{q}$ satisfies the differential equation:

$$\dot{\mathbf{q}} = [\mathbf{C}'\mathbf{AL}_2 - \mathbf{KCAL}_2]\mathbf{q} + [\mathbf{C}'\mathbf{AL}_2\mathbf{K} + \mathbf{C}'\mathbf{AL}_1 - \mathbf{KCAL}_1 - \mathbf{KCAL}_1\mathbf{K}]\mathbf{y} + [\mathbf{C}'\mathbf{B} - \mathbf{KCB}]\mathbf{u}.$$

The reconstructed system state vector looks like:

$$\hat{\mathbf{x}} = \mathbf{L}_2\mathbf{q} + (\mathbf{L}_1 + \mathbf{L}_2\mathbf{K})\mathbf{y}. \quad (3)$$

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The equations (1) and (3) describe the reduced order observer. To convert a continuous observer in discrete assume that

$$\dot{\mathbf{x}} \approx \frac{\mathbf{x}_n - \mathbf{x}_{n-1}}{T},$$

where T is a sampling time.

After we restored the state vector, we can synthesize control law (in which it is suggested that the full state vector is known), replacing true state with the restored one.

Regulator uses output stationary feedback

$$\mathbf{u}(i) = -\mathbf{F}\hat{\mathbf{x}}(i),\ i = i_0, i_0+1, ..., i_1-1,$$

where **F** is the gain coefficients for every variable of state vector.

Meanings of these coefficients for the expression are calculated by the formula:

$$\mathbf{F} = \left(\mathbf{B}_d{}^T\mathbf{P}\mathbf{B}_d + \mathbf{R}_1\right)^{-1}\mathbf{B}_d{}^T\mathbf{P}\mathbf{A}_d. \quad (4)$$

In the expression (4) $\mathbf{P}$ - is a positively defined symmetrical matrix. It is a solution of equation:

$$\mathbf{P} = \mathbf{A}_d{}^T\mathbf{P}\mathbf{A}_d - \mathbf{A}_d^T\mathbf{P}\mathbf{B}_d\left(\mathbf{B}_d^T\mathbf{P}\mathbf{B}_d + \mathbf{R}_1\right)\mathbf{B}_d^T\mathbf{P}\mathbf{B}_d + \mathbf{R}_2.$$

Thus the optimal control law is a combination of reduced order deterministic observer, in which the systems state is restored. And the optimal deterministic controller, that is immediate linear function of restored state vector. This result is knows as the separation principle in implicit form [1], [3].

The next stage is the robastization of the obtained control system [2]–[4].

The last step in the procedure of the robust control system synthesis is a modelling system in Simulink package with all the necessary nonlinear elements belonging to a real system (saturation, dead zone, etc.), and the turbulent wind. Thus, the final conclusion about the control system quality can be done after modelling [2], [3].

III. Case study

Consider the lateral motion of a small UAV. In accordance to methods, which are described above, structural parametric synthesis of robust control system consists of three stages:

1. Synthesis of the reduced order estimator.
2. Synthesis of the optimal regulator. Robust optimization of obtained optimal result.
3. Closed loop system modelling.

On the first stage of the synthesis of the optimal control system, restoring of the full vector of the system state is performed. It was made with help of reduced order estimator.

The object stat space vector is $x = [\beta \quad \varphi \quad p \quad r \quad \psi]$: β is the yaw angle; φ is the roll angle; p is the roll rate; r is the yaw rate; ψ is the rate. Synthesis is performed for the series connection of the actuator and the plant. So the resulting system order is 6. Only for variables are measured: φ, p, r, ψ.

For robust optimisation tow model was considered: nominal (air speed 250 km/h) and perturbed (air speed 200 km/h).

At first the reduced-order observer was synthesized. It is necessary to find the observer eigenvalues so that they minimize the matrix condition number $\bar{C}$ [5]. The matrix condition number minimum does not provide optimum quality of the connected with the observer object. Therefore, penalty function and H_2-norm is includes in the performance index that minimizes. So we provide optimality of the reduced order observer.

The initial vector of reduced-order observer eigenvalues equals

$$\mathbf{Pk} = [-4.8120 \quad -0.02387 \quad -4.8120 \quad -2.7416]$$

The result of the optimization procedure is

$$\mathbf{Pk} = [-0.0541 \quad -1.0248 \quad -7.3573 \quad -0.1682]$$

The matrix condition number equals 1.08583. It provides sufficient quality of the state space vector restoration. But such coefficient does not provide the maximum quality of the object connected with the observer. Therefore, the optimization procedure is performed in order to minimize the H_2-norm, as the performance of the system. The initial data for the optimization procedure are the eigenvalues obtained in the previous step. During the optimization procedure the following eigenvalues of the reduced order observer was obtained:

$$\mathbf{Pk} = [-0.0251 \quad -11.9995 \quad -6.8112 \quad -0.0527].$$

Using the reconstructed state vector of the system it is possible to implement the linear control low. For this purpose it is necessary to specify the coefficients $\mathbf{R}_1$ and $\mathbf{R}_2$ (4). In this case they are following:

R1=[1.2 0.1 15 75 0.001 0.01];

R1=diag(R1); **R2**=5.

As a result, the gain factor is obtained

$$\mathbf{F} = \left[0.3039 \quad 8.4524 \quad -6.7894 \quad -2.6904 \quad -0.0118 \quad 8.3230\right].$$

Eigenvalues **Pk** of the reduced order observer and the regulator gain **F** are the initial data for robust optimization procedure.

Parameters that satisfy specified requirements obtained with the following values of the weight coefficients in the composite performance index [2], [3]:

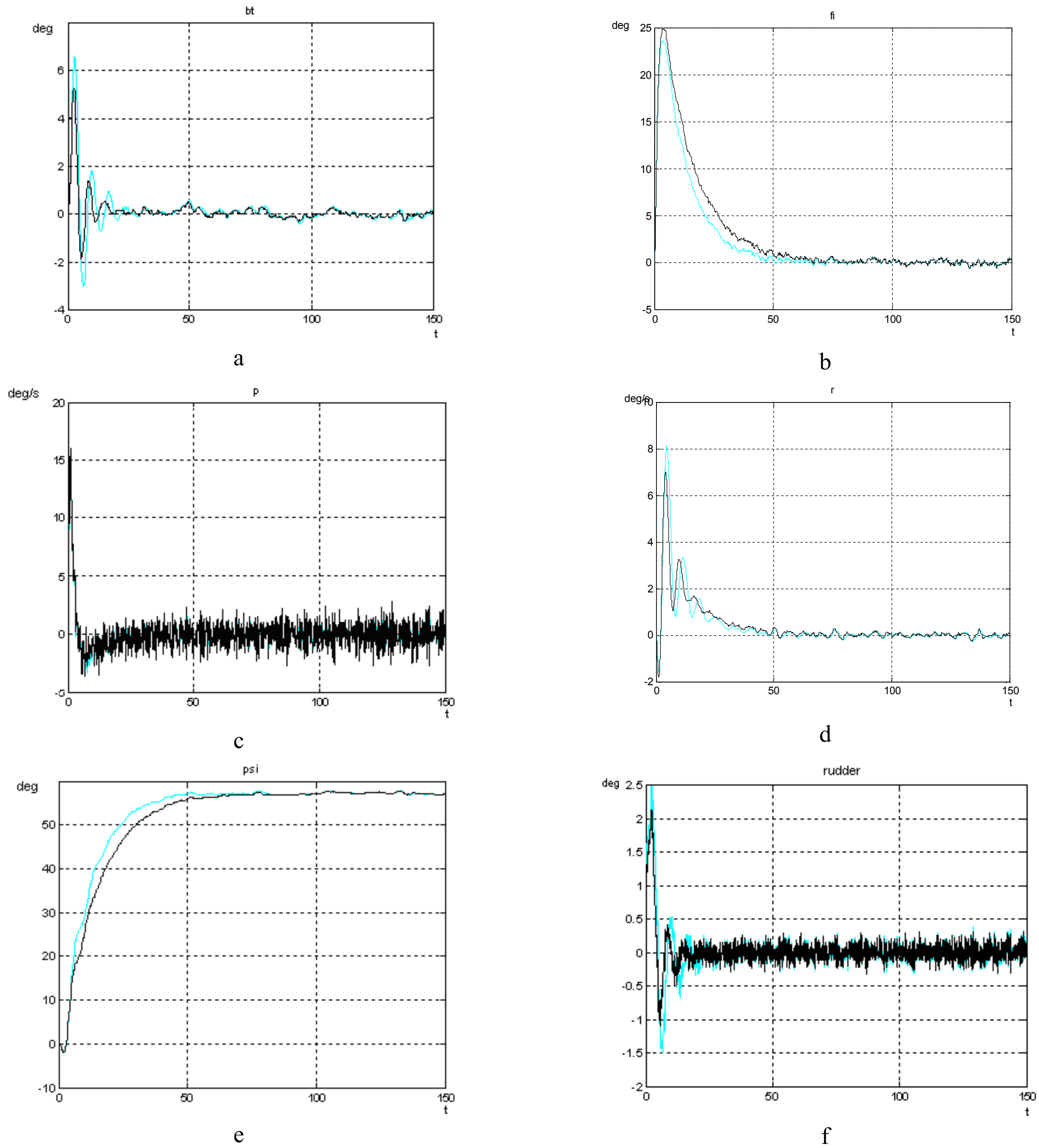
$$\lambda_{sn} = \lambda_{sp} = 24.5,\ \lambda_{dn} = \lambda_{dp} = 1,8,,\ \lambda_{\infty} = 7.25.$$

The synthesized controller is described by matrices $[\mathbf{A}_r \quad \mathbf{B}_r \quad \mathbf{C}_r \quad \mathbf{D}_r]$ that are following:

$$\mathbf{A}_r = \begin{bmatrix} 0.9947 & 0 \\ 0.0013228 & 0.9943 \end{bmatrix}; \quad \mathbf{B}_r = \begin{bmatrix} 0.002799 & 0.0004628 & -0.019998 & 0 \\ -0.02356 & -0.02308 & -0.0006244 & -0.01159 \end{bmatrix};$$

$$\mathbf{C}_r = [0.03322 \quad -0.1068]; \qquad \mathbf{D}_r = [-0.5895 \quad -0.4504 \quad 0.00118 \quad -0.29].$$

Performance and robustness indices are given in the Table. The result of perturbed and nominal closed loop systems simulation represented in Figure. Modelling performed for system with turbulent wind, the standard deviation of the instantaneous velocity is equal to 2.5 m/s.



Simulation result for nominal (black line) and perturbed (gray line) systems: a is a sideslip angle; b is the roll angle; c is the roll rate; d is the yaw rate; e is the yaw angle; f is the rudder deflection

VALUES $\mathbf{H}_2$ - AND $\mathbf{H}_\infty$ -NORM FOR NOMINAL AND PERTURBED OPTIMAL AND ROBUST SYSTEMS

Object		Optimal	Robust
$\mathbf{H}^s{}_2$	nominal	2.1015	2.1446
	perturbed	2.5516	2.3844
$\mathbf{H}^d{}_2$	nominal	0.8942	2.0636
	perturbed	0.6222	1.8471
$\mathbf{H}^d{}_\infty$	nominal	2.4197	0.7062
	perturbed	1.9403	0.5660

CONCLUSIONS

In Table it is shown that robustness and performance in stochastic case for robust system is better then performance of optimal system. To achieve such a result the performance in deterministic case was reduced. Simulation results and $\mathbf{H}_2$ -, $\mathbf{H}_\infty$ -norms demonstrate that flight requirements are held. But the performance of the robust control system in the deterministic case worse then the performance of the optimal control system. On the other hand the difference between values of all performance indices for nominal and perturbed models of UAV with the robust control system is smaller then with the optimal control one.

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Т. А. Галагуз. Структурно-параметричний синтез робастної системи керування зі спостерігачем пониженого порядку

Розглянуто особливості синтезу системи керування зі спостерігачем пониженого порядку. Синтезовано оптимальний закон керування та виконано робастну оптимізацію. Результати отримано для бічного каналу малого безпілотного літального апарата.

Ключові слова: безпілотний літальний апарат; оптимальний закон керування; робастне керування; спостерігач пониженого порядку.

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Т. А. Галагуз. Структурно-параметрический синтез робастной системы управления с наблюдателем пониженного порядка

Рассмотрены особенности синтеза системы управления с наблюдателем пониженного порядка. Синтезирован оптимальный закон управления и выполнена робастная оптимизация. Результат получен для бокового канала малого беспилотного летательного аппарата.

Ключевые слова: беспилотный летательный аппарат; оптимальный закон управления; робастное управление; наблюдатель пониженного порядка.

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